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USER MANUAL FOR PROGRAMS VISICE AND VISFIT.(U)

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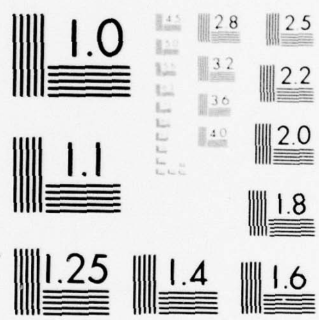

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CIVIL ENGINEERING LABORATORY
Naval Construction Battalion Center
Port Hueneme, California

Sponsored by
NAVAL FACILITIES ENGINEERING COMMAND

USER MANUAL FOR PROGRAM VISICE AND VISFIT

June 1979

An Investigation Conducted by

UNIVERSITY OF NOTRE DAME
Notre Dame, Indiana

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Study, transforms an original finite element computer program into a simplified user-format that is readily useable by design and field engineers to determine the structural capacity of a floating sea ice sheet to surface loads. Included with the program development is an instruction manual that provides step-by-step instruction for data input and output interpretation. The

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Block 20. Abstract (Cont'd)

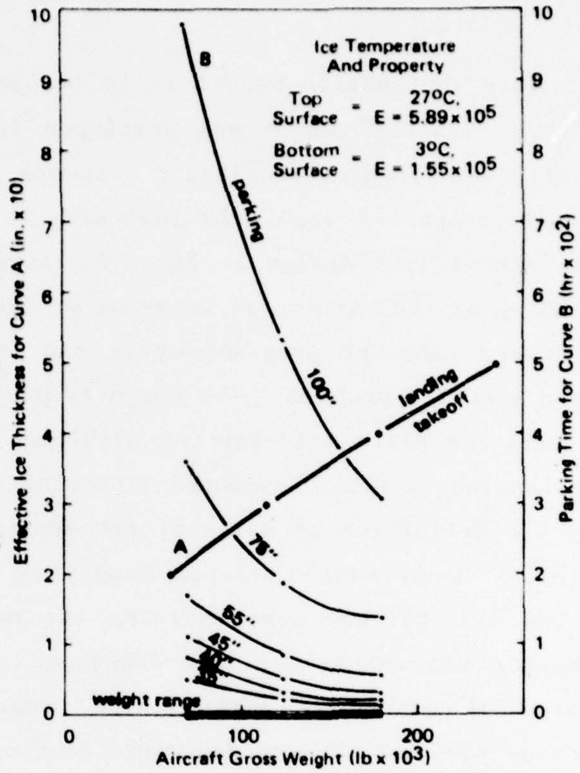
computer program permits solving problems of elastic and visco-elastic description.

FOREWORD

This instruction manual is to be used with finite element computer Program "VISICE," which was developed specifically for predicting the elastic and linear viscoelastic response of a floating sea ice sheet to a surface-applied load. By following the step-by-step instructions in this manual, the design or field engineer with a reasonably good understanding of the structural behavior of sea ice can readily formulate the necessary computer program inputs and interpolate the computed output for a specific problem. The computer program provides three methods for solving ice plate load-bearing problems: (1) using the Civil Engineering Laboratory (CEL) developed viscoelastic material model together with the CEL definition of material properties, (2) using CEL linear elastic material model with the user supplying definition of mechanical properties, or (3) the user defining the material model for viscoelastic behavior and associated time-dependent sea ice creep properties. The choice of method is left to the program user. The procedures for methods (1) and (2) are described as Program VISICE in the main body of the manual and for method (3) in Program "VISFIT," Appendix B.

The user of the VISICE program can limit the solution to elastic behavior response to short-term loading, for which stress criteria dictate the allowable load; or the program can include viscoelastic behavior response, for which deflection criteria dictate allowable loading time. Generally speaking, both types of information are useful to the field operator. For aircraft operation, the results can be conveniently plotted as a set of curves, an example of which is shown in Figure F1 for the C130 aircraft. The data for the ice-thickness-versus-equipment-weight curve, which represents elastic behavior, and for the load-time-versus-equipment-weight curve, which represents linear viscoelastic behavior, can be obtained by executing one set of computer runs for which the ice plate thickness is varied while the equipment weight is held constant. Since stress and deflection are linearly proportional to the applied load, the remaining coordinate points to generate the curves (Figure F1) are derived by the ratio factor of original weight to a new weight for that equipment. Applying appropriate safety factors also enters into calculating the curve data points (e.g., allowable

stress for elastic behavior and allowable deflection for viscoelastic behavior). The curve-plotting format for loading time data (Figure F1) differs from that suggested in the body of the manual. Characterizing the data by this format seems to favor easier interpolation for the field operator.



A – Ice sheet thickness (minimum) versus aircraft gross weight for landing and takeoff

B – Parking time (approximate) versus aircraft gross weight until deflection equals 10% of ice thickness

FIGURE F1. EXAMPLE OF LOAD CURVES FOR A CARGO AIRCRAFT – THERMAL PERIOD 1

To assist the program user in estimating computer costs, a viscoelastic solution with print-out for one ice plate thickness and run on a C.D.C. CYBER 175 computer will consume approximately 700 system seconds. The problem is defined as: (1) ice plate with top-to-bottom temperature gradient; (2) superposition of six load points; (3) duration of load, 15 time-steps. For elastic solution only, computer time is reduced approximately 65%.

As a closing comment before using the information presented in the manual, the user should judge the viscoelastic model, material property values, and suggested safety criteria for adequacy in representing the conditions of the problem under consideration. It is well-known that sea ice can behave as a viscoplastic material when very long-term loads or loads of heavy weight are involved. It should also be kept in mind that the material properties of sea ice are environment dependent and therefore highly variable.

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User Manual for Programs VISICE and VISFIT

CONTENTS

INTRODUCTION

Background: Since 1970 CEL has developed two finite element programs for predicting the structural capacity of sea-ice sheets subjected to aircraft and ground vehicle loadings.

The first program, called WILICE (1), provided an elastic representation of the sea-ice, and the second program, called VISICE (2), provided a viscoelastic representation which inherently includes elastic analysis as a special case. Thus in theory, the VISICE program completely supplants the WILICE program because it contains the same analytical capabilities in addition to providing viscoelastic analysis.

The analytical approach considers the sea-ice sheet as a large axisymmetric solid (plate-like geometry) resting on a fluid foundation. The sea-ice cross section is modelled with four node isoparametric elements with viscoelastic properties. Initially, loading is considered for a single pressure disc at the axis of revolution. Thereafter, the single pressure disc solution is used as a data base for superimposing stresses from a specified pattern of disc pressures simulating aircraft or other vehicles.

The principle drawback of the original VISICE program is defining input. For example, bulk and shear relaxation functions to characterize sea ice at various temperatures are difficult to determine even for engineers highly knowledgeable in the field of viscoelasticity. Another area requiring a degree of expertise is proper finite element modeling, such as, mesh density and boundary conditions. Other input problems include tedious data preparation for superposition of wheel loads for standard Navy aircraft.

These and other input problems are addressed in the following objective as well as improving computer output and evaluating results.

Objectives: The objective of this work is to modify the original VISICE program into a user-oriented program readily useable by design and field engineers to rapidly determine structural capacity of sea-ice sheets for either elastic or viscoelastic description of sea-ice.

Scope and Approach: To meet the above objective, extensive modifications and revisions to the pre- and post-processing portions have been made. Specifically:

(1) Geometry Automation: A finite element mesh generation scheme has been modified and extended to automatically provide (at the user's option) plate extent to represent infinite plate, boundary conditions, sea water foundation, element density pattern, and arbitrary temperature profile through ice thickness. As a minimum, the user only needs to specify plate thickness and temperature at top and bottom of ice sheet.

(2) Material Properties: Tabularized material properties for sea-ice as a function of temperature are "built in" the program. The user need only specify "elastic" or "viscoelastic" and the appropriate properties are automatically assigned to each element. Alternatively, the user may specify his own temperature dependent properties. Appendix D provides a program for obtaining viscoelastic relaxation functions from creep data.

(3) Loading and Superposition: A library subroutine for common Navy aircraft (C121, C124, C130, C141, and C5) allows the user to simply identify the craft by name and percent of maximum weight. Superposition of all wheel loads are automatically provided. Alternatively, non-standard load patterns can be defined with simple input data.

The above features are discussed in detail in this report along with assumptions and limitations of the program. In the last section a discussion on evaluation of sea-ice capacity is presented. Throughout this report the discussion is concerned with applications and evaluation, no detailed finite element formulation is presented as this is given in Reference (2).

Appendix A provides input instructions along with example problems for the new VISICE program. Detailed notes are given with the input instructions, however, the user should read the main body of this report prior to exercising the program.

Appendix B gives discussion and input instructions for program VISFIT to convert creep data into viscoelastic relaxation functions which in turn can be used as input to VISICE. Normally it is recommended to use the "built in" sea-ice properties contained in VISICE.

Lastly, the program VISICE may be used to analyze general visco-elastic, axisymmetric solids. Input instructions for this mode of operation is given in Reference (2).

DISCUSSION OF VISICE PROGRAM

General Nature of Problem: The basic problem under consideration may be viewed as a large plate-like structure supported on a fluid foundation subjected to a set of surface loads as suggested in Figure 1. Surface loads are composed of circular disks of arbitrary pressure whose centroids may be arbitrarily located on the surface. However, the radius of each disk is the same. The plate-like structure is considered isotropic elastic or viscoelastic with arbitrary variations of material properties through the plate thickness.

Specifically, the physical interpretation of this problem is a vast sheet of sea ice floating on sea water with surface loads due to stationary aircraft, parked vehicles, or other equipment placed on the surface. It is assumed the sea-ice sheet behaves as an isotropic viscoelastic or elastic solid wherein the material properties may vary through the thickness due to temperature variations, and/or age effects of various strata.

The objective is to determine the adequacy of the sea-ice sheet to support the given loads over a specified time interval. This may be assessed by considering the maximum tensile bending stresses occuring at the bottom of the ice sheet as well as considering allowable vertical deflection. These notions will be discussed in subsequent sections.

Overview of Analysis Procedure: To analyze the class of problems outlined above, the VISICE program employs an axisymmetric finite element representation of the sea-ice plate supported on vertical springs which provide a vertical restoring force distribution proportional to the weight of displaced fluid as the plate deforms. Initially, loading is only considered for a single circular pressure disc at the axis of revolution. If the sea-ice plate is assumed viscoelastic a sequence of time step solutions are obtained providing a history of deformations. For the elastic case, only one solution step is required. In either case, displacements and stresses throughout the sea-ice plate are saved on slow speed storage for each time step. After the single load solution is complete, the program shifts into the load-superposition

mode to calculate combined stresses and displacements at specified points due to a specified load pattern. This is achieved by recalling the appropriate stresses and displacements from storage, scaling them to specified pressure, transforming them to a common three dimensional coordinate system, and accumulating the results for each time of interest.

To initiate the program, three general areas of input information are required; (1) geometry of plate including finite element mesh density and temperature profile through plate thickness, (2) material properties of sea-ice as a function of temperature (either elastic or viscoelastic), and (3) magnitude and location of surface loads relative to points of superposition. Each of these areas are discussed below along with the automated features of the program that greatly reduce the labor of defining input.

Geometry: The fundamental dimensions of the plate is thickness (to be specified by user) and radius. In general, one perceives the sea-ice plate to be of infinite radius, however by the nature of the finite element formulation, a finite radius must be selected. The goal is to select a radius sufficiently large such that the boundary conditions on the plate periphery have negligible influence on the single load solution. At the same time, it is not prudent to select a radius excessively large because many additional elements would be required and increase computational costs. Numerical experimentation has shown that a radius, RMAX, on the order of 10 times the radius-of-relative-plate-stiffness, ℓ, is generally adequate, i.e.,

$$\mathrm{RMAX} = 10.\ell \qquad (1)$$

$$\ell = (Eh^3/12(1-v^2)k)^{\frac{1}{4}} \qquad (2)$$

where E = Young's initial elastic modulus, h = plate thickness, v = Poisson's ratio, and k = fluid weight density. In the VISICE program RMAX is automatically computed as a default option. A simple method of checking the adequacy of RMAX is to change the boundary conditions on the plate periphery. Generally it is assumed displacements on the periphery are fixed.

Within the plate cross-section a finite element mesh is automatically established by specification of three parameters (NUMELZ, NELOAD, and NUMELR) as shown in Figure 2. NUMELZ is the number of element rows evenly spaced through the plate thickness. NELOAD is the number of element columns evenly spaced directly beneath the single load radius, and NUMELR is the total number of element columns in the radial direction. Thus the total number of elements is the product NUMELZ times NUMELR. The column elements between the loading radius and outer plate radius (NUMELR-NELOAD) are spaced in a geometrically increasing fashion for the purpose of maintaining a high element density in the central part of the plate where stress gradients are highest.

As with any finite element model, the more elements used the better the approximation, but also, the higher the computational costs. Typical mesh parameters are; NUMELZ = 8, NELOAD = 5, and NUMELR = 25. It is good practice to test at least two mesh density patterns and verify the solutions are acceptably close.

The temperature profile through the plate thickness must be specified in accordance with the temperature distribution in the sea-ice sheet under investigation. This is accomplished by defining a series of arbitrary points through the plate thickness at which the temperature is specified. The sole purpose of this temperature profile is to provide a data base for the VISICE program to automatically determine temperatures at element centers by linear interpolation. Thereafter, temperatures at element centers are used to obtain temperature dependent material properties (elastic or viscoelastic) from material data tables discussed next.

Material Properties: It is well known that natural sea ice is a crystaline structure with orthotropic properties, i.e., its stiffness in the vertical direction is greater than the radial direction. However for the type of boundary value problem being considered, an isotropic assumption is reasonable providing the stiffness and/or creep properties in the radial direction are used. This is because the primary mode of deformation is plate bending so that strain energy

is predominately due to radial and tangential strains, not vertical strains.

With the above understanding, the VISICE program offers two isotropic characterizations of the sea-ice, elastic or viscoelastic. Actually, the elastic solution is a subset of the viscoelastic solution given at time = 0. However for convenience and simplicity, the input for elastic properties are given separately from viscoelastic properties. In either case, the material properties are input in tabular data form as a function of temperature, either as specified by the user (LEVEL = 1), or automatically provided the program as an alternative option (LEVEL = 0). Using this tabular data and linear interpolation, the program assigns material properties to each element row corresponding to temperatures at element centers as determined from the temperature profile.

For user supplied elastic properties (LEVEL = 1), the user specifies; temperature, Young's modulus, and Poisson's ratio for as many temperatures as desired. Alternatively for the automatic option (LEVEL = 0), the program supplies the data shown in Table 1 which is taken from Reference (3).

For user supplied viscoelastic properties (LEVEL = 1), the user must define parameters for bulk and shear relaxations functions for as many temperatures as desired. These functions have form:

$$K(t) = K_o + \sum_{i=1}^{N_B} K_i e^{-t/\alpha i} \tag{3}$$

$$G(t) = G_o + \sum_{i=1}^{N_G} G_i e^{-t/\beta i} \tag{4}$$

where K_o, K_1, . . . K_{N_B} are bulk modulus coefficients with the units of stress and α_1, α_2, . . . α_{N_B} are bulk relaxation times associated with a particular temperature. Similarly, G_o, G_1, . . . G_{N_G} are shear modulus coefficients with untis of stress, and β_1, β_2, . . . β_{N_B} are shear relaxation times.

Determination of these parameters from creep experiments is not a trivial task. Appendix B provides a discussion along with a special purpose program called VISFIT to convert creep data into relaxation functions.

Fortunately, the automatic viscoelastic input option (LEVEL = 0) provides an expedient alternative to defining relaxation function parameters by making use of predetermined relaxation functions developed from experimental work by Vaudry (3). Table 2 shows the tabular form of these viscoelastic parameters for four temperatures and is automatically supplied by the program if requested. Here it is assumed that Poisson's ratio is constant, $v = 0.3$, so that bulk and shear relaxation times are identical and the modulus coefficients are related through Poisson's ratio in the same manner as an elastic solid. Note two exponential terms are defined for both bulk and shear but the constant terms K_o and G_o (modulus values at time = ∞) are set to zero. This implies the model is fluid-like and will continue to creep under sustained loading. For further insight into the nature of viscoelastic behavior see Appendix B.

Loading and Superposition: The radius of the pressure disc is the primary loading parameter for the axisymmetric solution which in turn provides the data base for subsequent superposition of specified patterns of pressure discs. Thus, although the relative locations of the pressure discs are arbitrary, they are inherently restricted to the same radius. Pressure magnitudes on each disc may be specified independently because the axisymmetric solution is linearly dependent on load, allowing the data base to be appropriately scaled prior to superposition. For viscoelastic solutions, it is assumed that each independent pressure magnitude occurs instantaneously at time = 0.0 and remains constant throughout the loading duration.

Superposition occurs at a point implicitly defined the surface coordinate locations of pressure disc centroids and explicitly defined by a verticle coordinate, Z_p. That is, a three dimensional Cartesian Coordinate system (X, Y, Z) is inferred with its origin on the plate

TABLE 1. ELASTIC PROPERTIES OF SEA-ICE (REF. (3))

Temperature °C	Young's Modulus psi x 10^5	Poisson's Ratio
-27.0	5.89	0.30
-20.0	3.94	0.30
-10.0	2.42	0.30
- 4.0	1.55	0.30

TABLE 2. VISCOELASTIC PROPERTIES OF SEA-ICE (REF. (3))

Temp. °C	K_1 * psi	α_1 hr.	K_2 psi	α_2 hr.	G_1 psi	β_1 hr.	G_2 psi	β_2 hr.
-27.0	4.72	4.50	0.19	2010.	2.18	4.50	0.087	2010.
-20.0	3.16	3.51	0.12	1350.	1.45	3.51	0.057	1350.
-10.0	1.93	3.58	0.089	2540.	0.89	3.58	0.041	2540.
- 4.0	1.23	4.10	0.056	2400.	0.57	4.10	0.026	2400.

* All modulus coefficients of bulk and shear are times 10^5. Note: $K_o = G_o = 0.0$.

surface as suggested in Figure 3. The point of superposition is at the location (0, 0, Zp) where Zp is generally selected at the plate bottom (i.e., Zp = -h) because tensile stresses are generally highest there. The relative locations of the pressure discs are defined by the surface distances Xi, Yi away from the point of superposition.

VISICE permits repeating load superposition input data as many times as desired to consider different points of superposition, or to consider completely different load patterns and/or magnitudes. If it is desired to get superimposed results at several locations for a constant load pattern, one must redefine the pressure disc coordinates Xi, Yi relative to each new superposition point. Usually, one is interested in specifying superposition points where tensile stresses and/or vertical displacements are maximum (not necessarily the same point). As a general guideline, maximum tensile stresses usually occur at the plate bottom directly beneath a pressure disc. If all pressure discs have nearly the same pressure, that pressure disc closest to the center of resultant loading is the most likely candidate. Maximum vertical displacement often occurs at the same surface location, or sometimes at the loading resultant center.

Circular pressure disc patterns may be used to construct a variety of load patterns and distributions. For example rectangular load patterns or other geometric shapes may be approximated by a sequence of overlapping discs as suggested in Figure 4. In cases like this, the total weight of the vehicle should be maintained by the pressure discs even if the pressure distribution is not exactly correct.

Since Navy aircraft parked on sea-ice sheets is the primary problem of concern, an automatic superposition data generator has been incorporated into the VISICE program in subroutine LDTYPE which may be used as an alternative option. Here, the location and pressure of each individual tire is automatically established for five different types of aircraft given in Table 3. The user need only identify the aircraft by name and the actual aircraft weight by specifying the fraction of the maximum allowable aircraft weight to be considered. The program automatically distributes the actual weight to each wheel. Tires on the main gear

TABLE 3. NAVY AIRCRAFT AND LOADING CHARACTERISTICS

Aircraft	Max. Wt. (lbs)	Number of Tires			Tire Print Radius (inches)	Weight Percent on each tire		Maximum* Tire-Disc pressure (psi)
		Total	Main	Nose		Main %	Nose %	
C121	145000.	6	4	2	8.48	23.88	2.25	153.0
C124	216400.	6	4	2	14.3	23.35	3.30	79.0
C130	135000.	6	4	2	10.4	23.95	2.15	95.0
C141	316000.	10	8	2	8.14	11.80	2.80	180.0
C5	635800	28	24	4	8.00	3.69	2.85	141.25

* This pressure is for tires on the main gears for aircraft loaded to maximum weight.

receive a larger percentage of the load than tires on the nose gear as shown in Table 3. This data was taken from Reference (4).

Figure 5 shows the relative tire print locations for each of the five aircraft (4). Two points of superposition are automatically established by the program denoted as points A and B if Figure 5. Point A is located at the intersection of the centerline and first main gear axis. Point B is shifted along the main gear axis to the center of the first tire. In both cases the Z coordinate of the superposition point is at the plate bottom. Generally Point B is the critical location for maximum tensile stresses and vertical displacements. For stiff plates, maximum vertical displacements may occur at Point A.

If desired, the user may easily modify subroutine LDTYPE to consider other points of superposition or extended the library to establish automated superposition for new aircraft and vehicles. Directions for modifying LDTYPE are given by comment cards in the subroutine.

Computer Output: All input data whether specified by the user or automatically constructed by the program is printed out with appropriate titles and should not cause any confusion. Temperature profile, material properties, loading parameters, and fundamental geometric parameters are always printed out for each problem. However, the user has the option of deleting the printout of the finite element mesh topology (e.g., nodal coordinates, element connectivity, etc.).

The results of the axisymmetric solution for a single pressure disc (prior to superposition) include displacements at each node point and stresses and strains at each element center. For viscoelastic solutions, this data is given for each time step. Again, the user has the option to delete printing of the single disc solution if he is only interested in superimposed results.

Results from load superposition include the net vertical displacement, the complete state of 3-dimensional stress (X, Y, Z system), and the principle stresses at the superposition point. For viscoelastic solutions this data is repeated for each time step, providing a time history of displacement and stress. Each superposition point is processed individually one at a time and the results are preceded by printed output showing the relative location and load magnitude of each pressure disc. For the automated aircraft superposition option, the pressure disc numbers correspond to the numbers in Figure 5.

Of primary interest is the maximum vertical displacement and the maximum tensile principle stress. These quantities are useful to evaluate the adequacy of the sea-ice sheet to sustain the given loading as discussed next.

SUMMARY AND EVALUATION

Evaluation Criteria: Although the VISICE program is a powerful analysis tool, the analytical results are only useful if there exists some failure criteria with which to compare. Unfortunately, the state-of-the-art for sea-ice failure criteria is quite primitive and warrants intensive research. A measure of stress alone does not appear adequate to define a failure criterion because laboratory specimens under constant stress states have been observed to fail as function of loading duration, i.e., after a long period of creeping (3). Perhaps strain energy density may offer a proper basis for defining a failure criterion.

However until a viable failure criterion can be established, the only alternative is to employ methods that seem to have worked well in the past. Two criterions presented by Vaudry (3) are; (1) maximum allowable tensile stress, and (2) maximum allowable displacement. These criterion may be written as:

$$\sigma_T < \sigma_{All.}(T) \qquad \text{(stress)}$$

$$W < 0.1\,h \qquad \text{(displacement)}$$

Here σ_T is maximum principle tensile stress in the ice sheet (predicted from VISICE), and $\sigma_{All.}(T)$ is the maximum allowable tensile stress dependent on sea-ice surface temperature as shown in Table 4. These allowable stresses were obtained from flexural strengths of sea-ice beams assuming a safety factor of 1.2 (Vaudry suggests safety factors between 1.15 and 1.20).

For the displacement criterion, W is the maximum vertical displacement in the ice sheet (predicted from VISICE) and h is the plate-thickness. Thus, the criterion states that maximum allowable displacement is 10% of plate thickness. One rational for this criterion is that deflections exceeding 10% h imply the ice surface has deflected below the free board water line allowing surface flooding through cracks and disrupting activities.

TABLE 4. ALLOWABLE TENSILE STRESSES

Thermal Period (Season)	Surface Temp. ^{o}C	Flexural Beam Strength psi	Allowable Stress* $\sigma_{All.}$ psi
1	-20 to -10	70.0	58.3
2	-10 to -5	62.0	51.7
3	-5 to -2	58.0	48.3
4	-3 to -2	40.0	33.3

* Based on safety factor of 1.2.

Applications: When applying these criterions to a specific problem involving a viscoelastic sea-ice representation, one observes maximum stress usually occurs immediately at time = 0.0 and then slightly decreases due to relaxation. Thus, one can immediately determine if the sea-ice sheet has sufficient thickness and strength (temperature) to initially carry the load.

If both stress and displacement criterions are initially safe (t = 0.0), sooner or later continued sea-ice creep will give displacements exceeding the allowable. The time at which this occurs tells how long the vehicle can remain parked in one location. For this reason it is recommended to always use the viscoelastic characterization rather than time-independent elastic properties.

Rather than analyzing each specific problem, a more general approach is to develop graphs or tables that specify the minimum allowable ice thickness required for a particular vehicle as a function of load duration for each thermal period. Figure 6 shows how such a graph might look. The flat portion of the graphs imply minimum thicknesses are controlled by initial stress and/or initial displacement and the curved portion implies displacement controls at some later times. If the ice thickness is known, the graphs may be used to determine how long the vehicle can remain parked in one location (if at all).

To construct such graphs is simple. Merely obtain a set of solutions over a range of ice thicknesses for each thermal period and apply the stress and displacement criterion. That is, consider a particular vehicle for a particular thermal period and solve the problem for several thicknesses of sea-ice. If the thickness is too small, one observes the stress criterion is immediately (t = 0.0) violated and thus it cannot support the vehicle for any length of time. As thicknesses increases, a minimum threshold thickness will be found that satisfies the stress criterion and the initial displacement criterion (i.e., the thickness associated with the flat portion of the curve in Figure 6). As loading duration continues for this threshold thickness, the displacement criterion will eventually be violated. This terminates the flat portion of the curve and indicates how long the vehicle may remain

parked on the minimum threshold thickness. For larger thicknesses, only the displacement criterion will control allowing longer loading durations as indicated the curved portion of the graphs in Figure 6.

In cases where the viscoelastic model is fluid-like (e.g., as provided by the VISICE program, Level = 1), the sea ice eventually begins to creep at a near constant rate as indicated in Figure B-1. Accordingly, it is not necessary to run the VISICE program throughout the entire loading history. Rather, it is more economical to terminate the program after the constant creep rate has been observed by noting displacement increments are near constant and stresses have reached a limiting value. Generally this occurs within 20 hours for models provided by the program. Thereafter, displacements may be hand calculated by linear extrapolation of the displacement increment over the remaining time of interest while stresses remain constant.

As a final comment, it is noted that all responses (stresses, strains, and displacements) are directly proportional to the total load magnitude distributed to all discs. Thus, the responses for different total load magnitudes may be scaled directly from any solution which only differs in load magnitude.

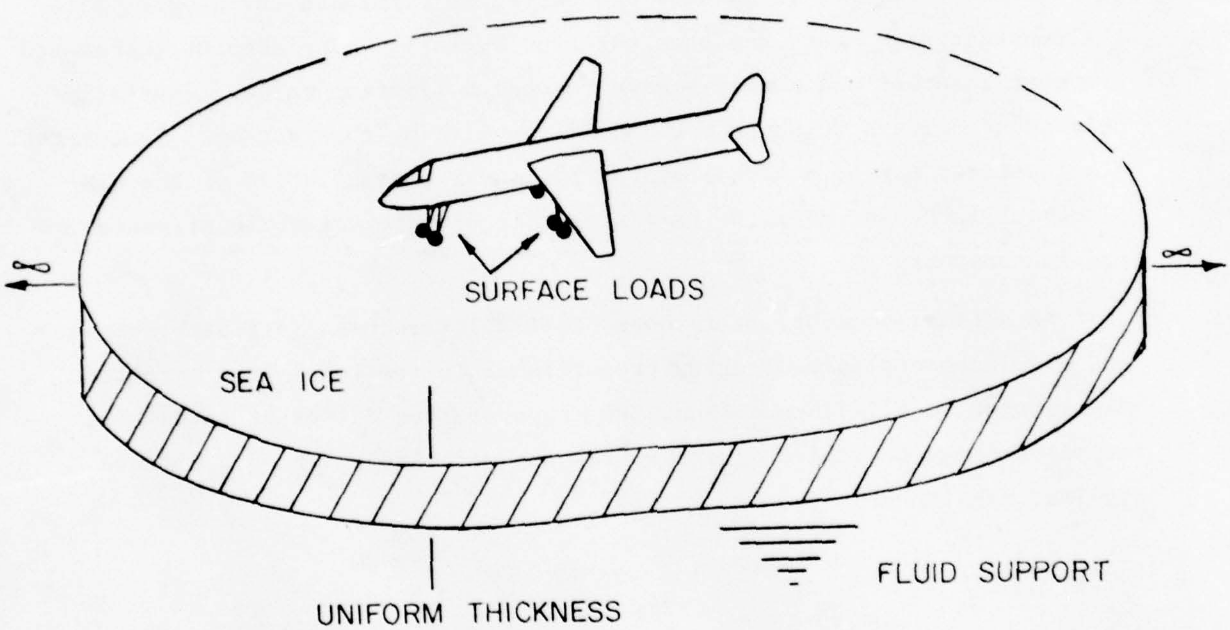


Figure 1. Representation of Boundary Value Problem.

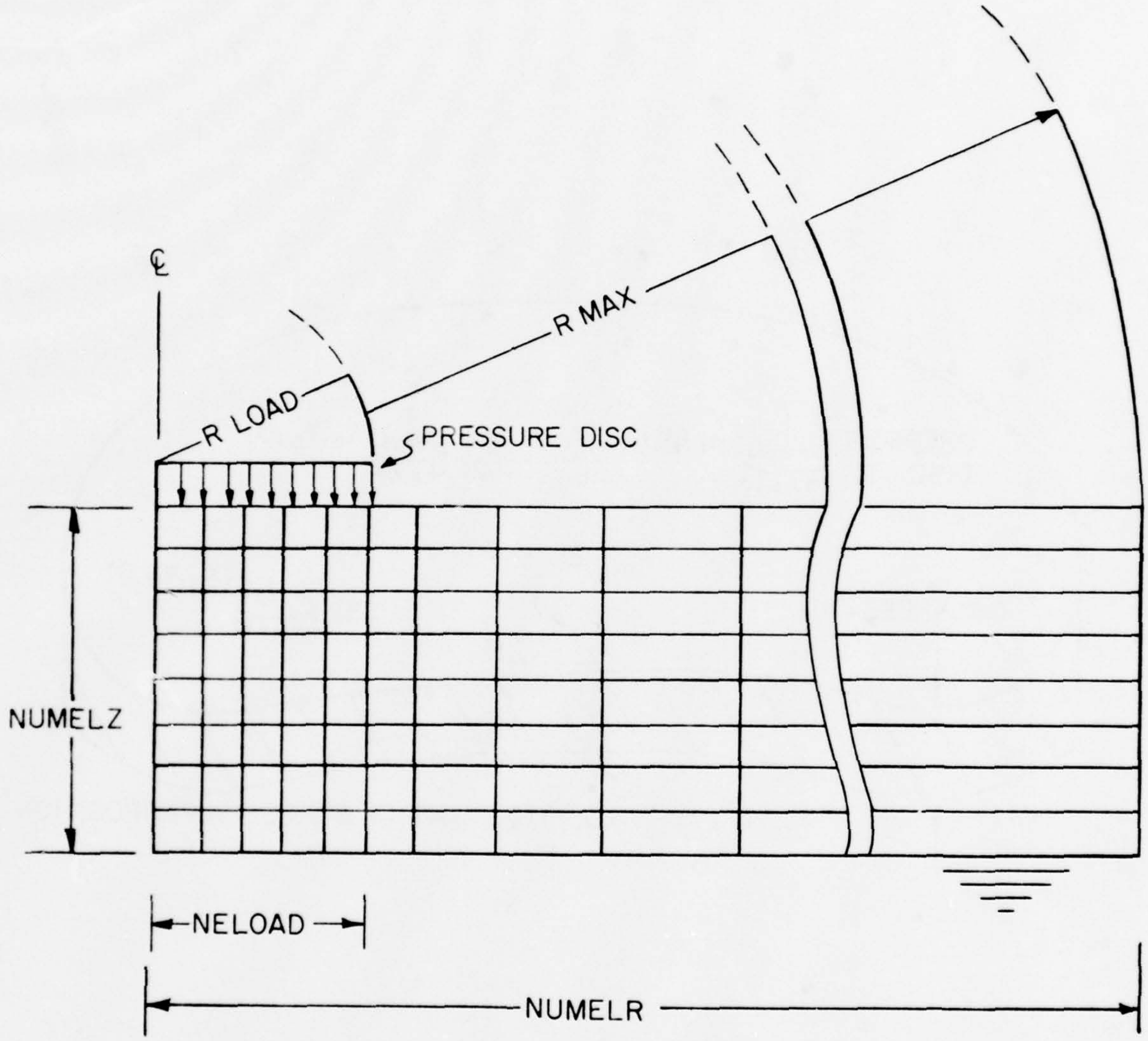


Figure 2. Cross-Section of Axisymmetric Plate-Solid with Finite Element Mesh Parameters.

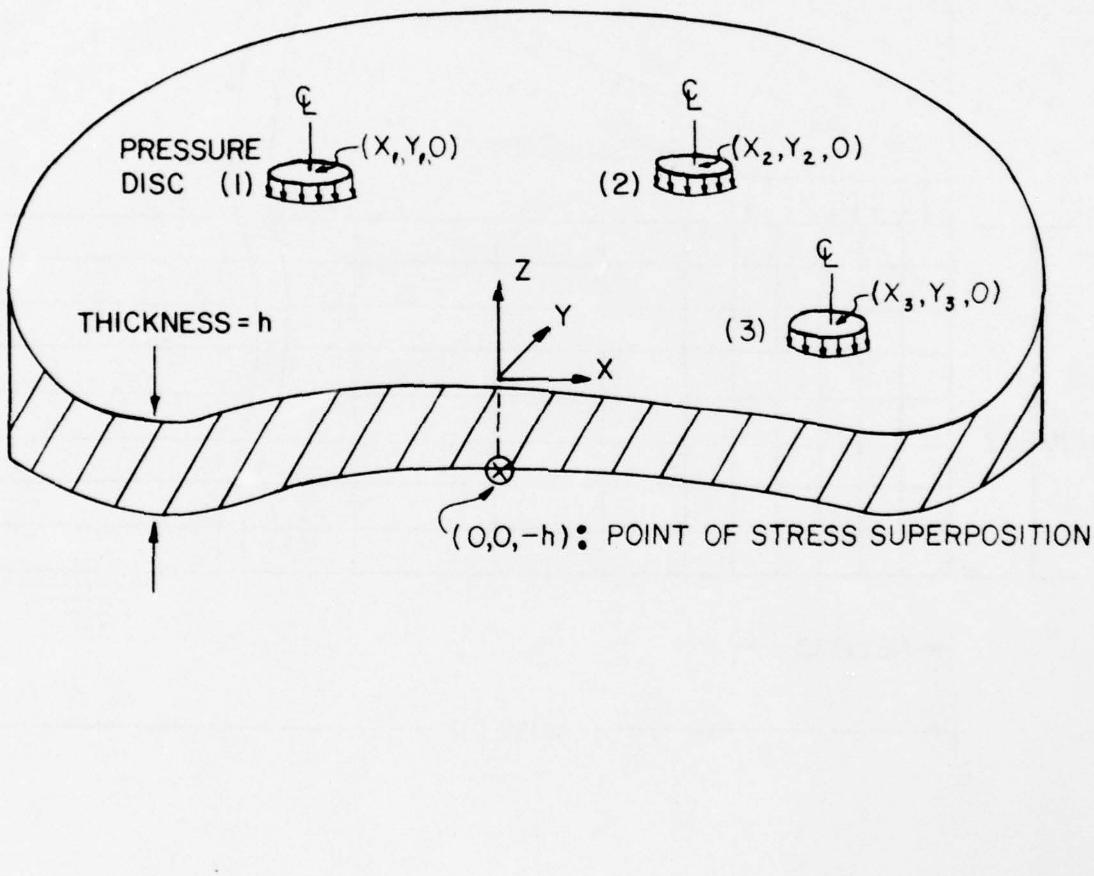


Figure 3. Pressure Disc Centroids Relative to Superposition Point.

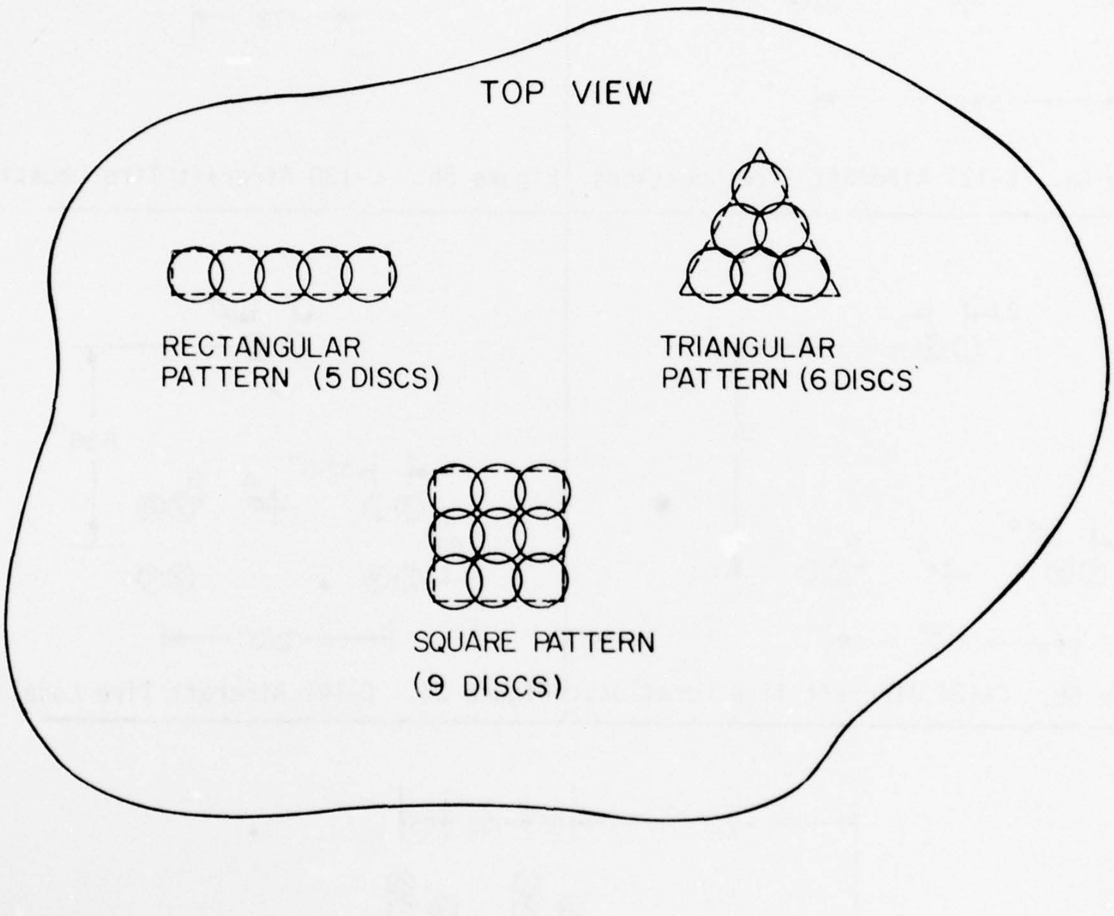
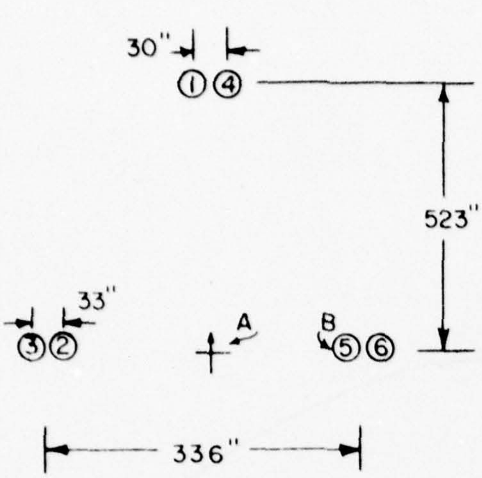


Figure 4. Disc Patterns to Approximate Geometrical Loading Shapes



Fiqure 5a. C-121 Aircraft Tire Locations.

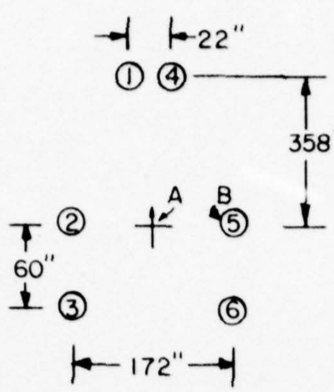
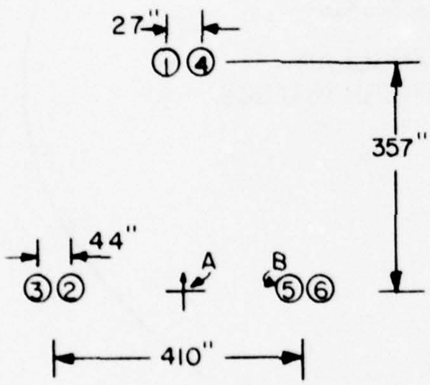


Figure 5b. C-130 Aircraft Tire Locations.



Fiqure 5c. C-124 Aircraft Tire Locations.

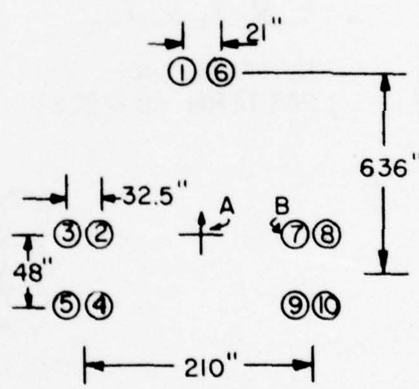


Figure 5d. C-141 Aircraft Tire Locations.

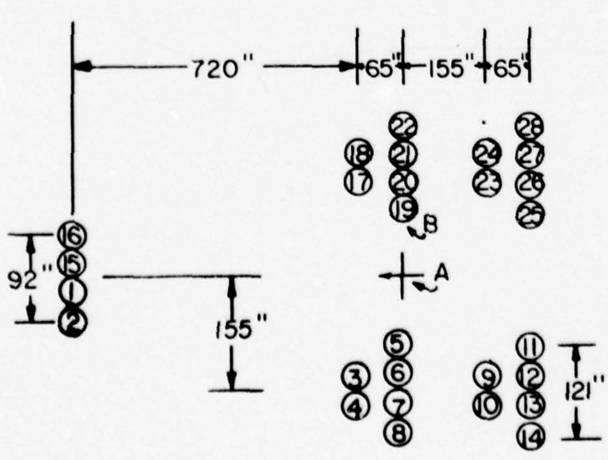


Figure 5e. C5 Aircraft Tire Locations.

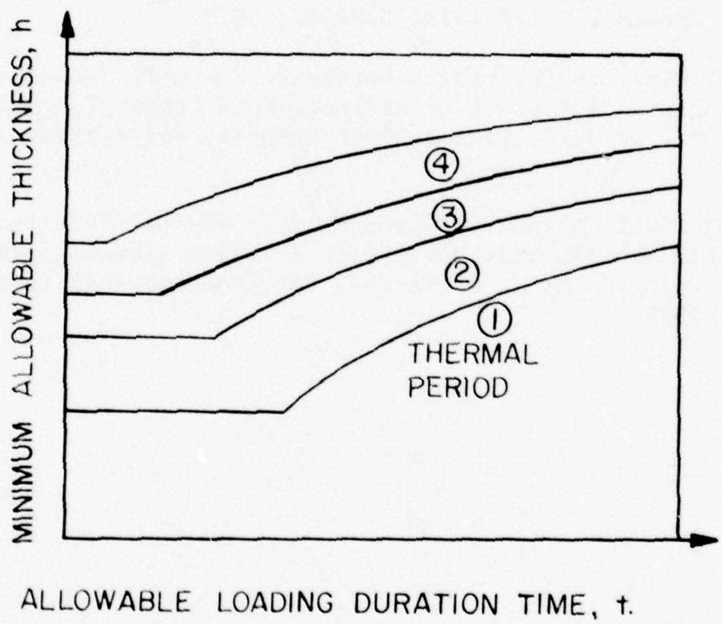


Figure 6. Hypothetical Graph of Vehicle-X, h-t Curves.

REFERENCES

1. Naval Civil Engineering Laboratory. TR R-797; "Ice Engineering: Summary of Elastic Properties Research and Introduction to Viscoelastic and Nonlinear Analysis of Saline Ice", by M. G. Katona and K. D. Vaudrey. Port Hueneme, California, August 1973.

2. Naval Civil Engineering Laboratory. TR-803; "Ice Engineering: Viscoelastic Finite Element Formulation", by Michael G. Katona. Port Hueneme, California, January 1974.

3. Naval Civil Engineering Laboratory. TR-860; "Ice Engineering: Study of Related Properties of Floating Sea-Ice Sheets and Summary of Elastic and Viscoelastic Analyses", by K. D. Vaudrey. Port Hueneme, California, December 1977.

4. Naval Civil Engineering Laboratory. TR-641; "Sea-Ice Bearing Strength in Antarctica - Aircraft Load Curves for McMurdo Ice Runway", by J. E. Dykins, Port Hueneme, California, September 1969.

5. Naval Civil Engineering Laboratory. TR-866; "A Viscoelastic-Plastic Constitutive Model with A Finite Element Solution Methodology", by M. G. Katona, Port Hueneme, California, June 1978.

APPENDIX

USER INPUT INSTRUCTIONS FOR VISICE AND EXAMPLES

This Appendix input provides instructions for data preparation to run the VISICE program. Following the input instructions are three example problems showing input data cards and selected computer output. It is assumed the user has read the main body of this report and is familiar with terminology, concepts, and assumptions.

Input data cards are grouped in six categories denoted by letters A through F shown below. For each problem begin with Card A1 and continue through to Card Set F.

Card Set A: Heading and Control
Card Set B: Vertical Plate Parameters and Temperature Profile
Card Set C: Material Properties and Time Parameters
Card Set D: Single Pressure-Disc Parameters
Card Set E: Radial Plate Parameters, and Output Control
Card Set F: Superposition of Loads

*** UNITS *** All units must be consistent. To be compatible with default options use:

Force	=	pounds
Length	=	inches
Pressure	=	psi
Temperature	=	centrigrade
Time	=	hours

*** DEFAULTS *** Default options automatically provide parameter input values whenever a blank (zero) data entry is encountered. Generally default options may be used, but it is prudent to cross-check default options as indicated in the subsequent notes.

PART I. INPUT INSTRUCTIONS

A. Problem Initiation, Heading, and Control Card

Card A1 (18A4)

Columns	Variable	Entry Description	Note
01-04	HED(1)	Control Word = MESH, call mesh generator = STOP, program stops execution ≠ STOP, or MESH, etc.	(1)
05-72	HED(1)	Descriptive title of the problem	(2)

- Go to Card Set B -

Notes:

(1) The control word MESH signals the automatic finite element mesh construction for the axisymmetric floating ice sheet with single pressure disc load. Problems may be run back to back as desired. To terminate execution use the control word STOP. (To access the more general finite element input option leave the control word blank, see CEL report TR-803 for input or program listing).

(2) Descriptive title is printed out with problem for user reference.

FORTRAN Reminders:

(1) Variables starting with the letters I, J, K, L, M, or N are integers (no decimal) and are right justified in allotted columns.

(2) Word variables (e.g. HED(1)) are left justified in allotted columns.

(3) Floating point variables may appear anywhere in allotted columns with decimal point.

B. Character of Ice-Sheet through Thickness, and Temperature Profile

Card B1 (F10.0,2I5)

Columns	Variable	Entry Description	Note
01-10	PTHICK	Thickness of the Ice Plate	(1)
11-15	NTEMP	Number of temperature points to define plate temperature throughout thickness Default = 1 Max. NTEMP = 12	(2)
16-20	NUMELZ	Number of evenly spaced Finite Elements rows through thickness Default = 8 Max. NUMELZ = 12	(3)

Card B2 (2F10.0) (Repeat card B2, NTEMP times)

Columns	Variable	Entry Description	Note
01-10	DEPTH(I)	Depth below surface where temperature is specified	(4)
11-20	TEMP(I)	Temperature in degrees centigrades	

- Go to Card Set C -

Notes:

(1) Ice mesh thickness is always input as a positive dimension (usually inches).

(2) For uniform temperatures set NTEMP = 1. For linear varying temperatures set NTEMP = 2. For more complicated temperature profiles use as many temperature points as required up to 12.

(3) Generally choose NUMELZ greater or equal to 6, see Figure 2.

(4) Depth is a positive number within the plate thickness. Repeat this card NTEMP times. If NTEMP = 1 any Depth may be used to specify uniform temperature. If NTEMP is greater than 1 the first temperature must be given at plate surface (Depth = 0) and the last temperature at plate bottom. Intervening temperature points must be given in order of increasing depth.

C. Material Properties (Temperature Dependent) and Time Parameters

Card C1 (A4, 1X, 3I5, F10.0)

Columns	Variable	Entry Description	Notes
01-04	TYPE	Word describing material type: = ELAS, Elastic Solution = VISC, Viscoelastic Solution ≠ ELAS or VISC, Error	(1)
06-10	LEVEL	Code for material properties input: = 0, Use canned properties of program = 1, User inputed properties	(2)
11-15	NUMT	Number of temperatures for which material properties will be given: Only for LEVEL = 1 Default = 4	(3)
16-20	NSTEP	Total number of time steps for TYPE = VISC only	(4)
21-30	DELT	Time step increment for TYPE = VISC only	(5)

If LEVEL = 0, go to card set D, otherwise go to card C2 for elastic input or card C3 for viscoelastic input.

Notes:

(1) This controls whether the sea-ice will be characterized as elastic or viscoelastic.

(2) For LEVEL = 0, the program automatically constructs (and prints out) temperature dependent elastic properties from Table 1, or viscoelastic properties from Table 2. For LEVEL = 1 the program will use temperature dependent properties from user input.

(3) This entry is only required for LEVEL = 1. The number of different temperatures at which elastic or viscoelastic properties will be defined.

(4,5) These entrys are only required for viscoelastic solution (LEVEL = 0 and 1). Total loading duration = NSTEP* DELT. Generally, choose the time step increment as 10 to 20% of shortest relaxation time. For LEVEL = 0, time steps in the range 0.5 to 1.0 hours are usually adequate. However, it is always wise to test accuracy by choosing smaller time steps and comparing results.

(Card Set C continued)

Elastic temperature properties data input. Input only if TYPE = ELAS and LEVEL = 1.

Card C2 (3F10.0) Repeat this card NUMT times.

Columns	Variable	Entry Description
01-10	XTEMP(N)	Temperature
11-20	XE(N)	Young's Modulus
21-30	XNU(N)	Poisson's Ratio

- Go to Card Set D -

Notes:

User defines tabular elastic properties (Young's modulus and Poisson's ratio) for the sea-ice at a set of discrete temperatures. Temperatures must be input in assending order (e.g., -27°, -20°, -10°, etc.). The program linearly interpolates material properties to each element based on the element's temperature at its center as determined from the temperature profile. If the element temperature is outside the range the tabular properties temperatures, the element is assigned properties from the tabular end points, i.e., data is not extrapolated.

(Card Set C continued)

Viscoelastic temperature property data input. Input only if TYPE = VISC and LEVEL = 1.

Card C3 (2I5)

Columns	Variable	Entry Description
01-05	NIB	Number of bulk exponential terms
06-10	NIG	Number of shear exponential terms

Card C4 (3F10.0)

Repeat cards C4, C5, and C6 as a group NUMT times (N = 1, NUMT)

Columns	Variable	Entry Description
01-10	XTEMP(N)	Temperature
11-20	XBKO(N)	Bulk modulus at infinity
21-30	XGKO(N)	Shear modulus at infinity

Card C5 (2F10.0) Repeat NIB times (J = 1, NIB)

Columns	Variable	Entry Description
01-10	XBK(N,2*J-1)	Bulk modulus of 'J' exponential
11-20	XBK(N,2*J)	Bulk relaxation of 'J' exponential

Card C6 (2F10.0) Repeat NIB times (J = 1, NIG)

Columns	Variable	Entry Description
01-10	XGK(N,2*J-1)	Shear modulus of 'J' exponential
11-20	XGK(N,2*J)	Shear relaxation of 'J' exponential

- Go to Card Set D -

Notes:

User defines tabular viscoelastic properties for sea-ice at a set of discrete temperatures in the same manner as discussed on previous page. Consult Appendix B for methods of determining relaxation function parameters.

D. Loading and Single Pressure-Disc Parameters

Card D1 (A4, 1X, 3F10.0)

Columns	Variable	Entry Description	Notes
01-04	CRAFT	Control word for load input: = USER, User supplies load data = C121, C124, C130, C141, or C5, automated input for Navy aircraft.	(1)
06-15	RLOAD	Radius of single pressure disc. (only required if CRAFT=USER)	(2)
16-25	PSI	Uniformly applied pressure on disc. (only required if CRAFT=USER)	(3)
26-35	PRATIO	Ratio of PSI applied Default = 1.0	(4)

- Go to Card Set E -

Notes:

(1) If CRAFT = USER the user defines his own load characteristics (and superposition parameters on Card F2). For Navy aircraft simply set CRAFT = name of desired aircraft to be analyzed, and all parameters are automatically defined as given in Table 3.

(2,3) This input is only required if CRAFT = USER. For Navy aircraft RLOAD and PSI are automatically established from Table 3. Note, PSI is set at maximum pressure of main gear tires to be used in single wheel analysis.

(4) PRATIO allows scaling the PSI on Navy aircraft as desired. For example, if the total actual weight of a C121 is 90% of maximum (Table 3), set PRATIO = 0.90.

E. Radial Plate Parameters and Boundary Conditions

Card E1 (2I5, 2F1.0, 3I5)

Columns	Variable	Entry Description	Notes
01-05	IN	Control for printed output of FEM mesh. = 0, Do not print ≠ 0, print output of FEM mesh	(1)
06-10	IOUT	Control for printed output of FEM results for single wheel load. = 0, Do not print ≠ 0, Print results	(2)
11-20	RMAX	Plate Radius Default = $347.0*(PTHICK)^{3/4}$ (inches)	(3)
21-30	GAMMA	Fluid weight density Default = 0.037 (lb/in^3)	
31-35	NELOAD	Radial elements under load Default = 5	(4)
36-40	NUMELR	Total radial elements Default = 25	(5)
41-45	KBOUND	Boundary code on plate periphery: = 1, Horizontally restrained = 2, Vertically restrained = 3, Completely fixed (Default)	(6)

- Go to Card Set F -

Notes:

(1,2) Finite element mesh data, and finite element solution for single wheel load may be printed out at user's option.

(3) The default plate radius is based on 10 times the radius-of-relative stiffness for initial ice stiffness at -27°C. Generally this is adequate to simulate infinite plate.

(4,5) These parameters control radial mesh density, see Figure 2. Generally the default options are adequate.

(6) Generally use the default option, KBOUND = 3, inplying fixed boundaries. Boundary conditions should not appreciably influence the solution for large plate radius.

F. Superposition and Load Pattern Data

Card F1 (I5)

Columns	Variable	Entry Description	Notes
01-05	INTSUP	Time interval of superposition: = 0, no superposition = 1, superimpose at each time step = N, superimpose at each nth time step	(1)

Card F2 (I5, 2F10.0) For CRAFT = USER only

Columns	Variable	Entry Description	Notes
01-05	NLOADS	Number of pressure discs to be be superimposed	(2)
06-15	PTOTAL	Total weight of all pressure discs (force)	(3)
16-25	ZPLANE	Vertical depth of desired superposition point (negative distance below surface)	(4)

Card F2 (3F10.0) For CRAFT = USER only. Repeat NLOAD times.

Columns	Variable	Entry Description	Notes
01-10	XLOC(I)	X location of pressure disc I center with respect to superposition point	(5)
11-20	YLOC(I)	Y location of disc I as above	(6)
21-30	PLD(I)	Fraction of PTOTAL assigned to disc I	(7)

If CRAFT = USER cards F2 and F3 may be repeated as many times as desired. To terminate set NLOADS = 0, and return to card A for new problem or to terminate program.

Notes

(1) Superposition is activated only if INTSUP $\neq$ 0. For superposition of elastic solutions always use INTSUP = 1. For superposition of visco-elastic solutions one can use INTSUP = 1 (generally recommended) or INTSUP = N wherein results will be printed for every nth time step.

(2,3,4) If CRAFT = USER, the user must supply the requested loading information. If CRAFT = (Navy aircraft) the user skips cards F2 and F3 and problem input is complete. Here the program automatically establishes load data from Table 3 where the total aircraft weight is scaled by PRATIO (Card D1). Superposition occurs at points A and B as shown in Figure 5.

(5,6,7) If CRAFT = USER, the surface coordinates of the pressure disc centers are defined relative to the desired point of superposition. Cards F2 and F3 may be repeated as desired to change point of superposition or to define new load distributions.

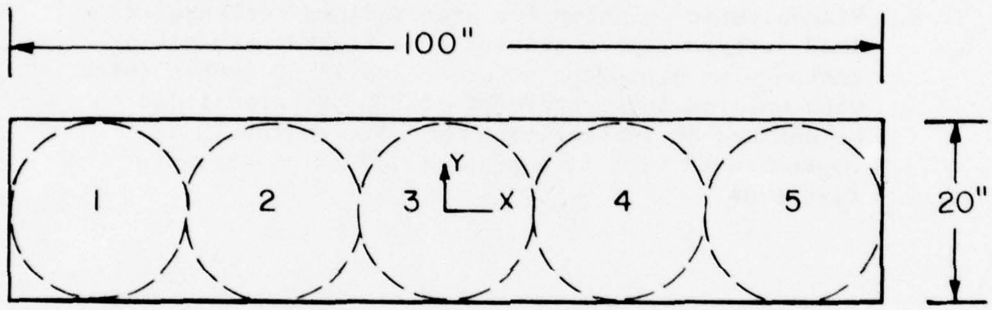
PART II. EXAMPLE INPUT/OUTPUT

Three example problems are shown with input card images and selected output to serve as benchmarks. Descriptions of the example problems follow:

1. Elastic solution for C141 aircraft on 66 inches of ice with linear temperature distribution -20^{o}C on surface to -4^{o}C on bottom. Superposition foroccur to 100% of maximum weight.

2. Viscoelastic solution for C130 aircraft on 66 inches of ice with linear temperature distribution of -27^{o}C to -3^{o}C, superposition for 90% of maximum weight.

3. Viscoelastic solution for user defined rectangular load pattern representative of a tracked vehicle or rectangular storage platform. Ice is 50 inches thick with unfirom temperature of -10^{o}C. Superposition to be defined at rectangle center. The rectangle is approximated with five pressure discs as shown in Figure A1.



TOTAL WEIGHT OF RECTANGULAR LOAD = 20,000 lb.

$$\text{PRESSURE ON EACH DISC} = \frac{1}{5}\,\frac{(20{,}000)}{\pi(10)^2} = 12.73 \text{ PSI}$$

Figure A1. Rectangular Load Pattern Approximation.

CARD INPUT IMAGES FOR PROBLEM 1.

```
COL. NO.  0    5    10   15   20   25   30   35   40   45   50   55

CARD NO.

     A1.  MESH  EXAMPLE 1.  C141 AIRCRAFT, ELASTIC SOLUTION.
     B1.          66.0     2
     B2.           0.0       -20.0
     B2.          66.0        -4.0
     C1.    ELAS      0
     D1.    C141                               1.0
     E1.        1     1
     F1.        1
     A1.  STOP
```

The above input data is the complete set of cards required for problem 1. On the following page is selected output from superposition at two points; C141 centerline axis, and under wheel closest to main axis.

Problem 1: Selected Output

FINAL SUPERIMPOSED DISPLACEMENTS AND STRESSES
FOR ALL LOAD CONTRIBUTIONS ARE GIVEN BELOW AT
EACH TIME STEP (ONE STEP FOR ELASTIC CASE)

- main axis -

TIME	VERTICAL DISP.	CARTESIAN STRESSES SIG(XX)	SIG(YY)	SIG(ZZ)	SIG(XY)	SIG(XZ)	SIG(YZ)	PRINCIPAL STRESSES SIG(1)	SIG(2)	SIG(3)
0.0	-0.130+01	0.260+02	0.410+02	-0.150+01	0.360-15	0.260-17	-0.170-04	0.410+02	0.240+02	-0.150+01

FINAL SUPERIMPOSED DISPLACEMENTS AND STRESSES
FOR ALL LOAD CONTRIBUTIONS ARE GIVEN BELOW AT
EACH TIME STEP (ONE STEP FOR ELASTIC CASE)

- wheel closest to main axis -

TIME	VERTICAL DISP.	CARTESIAN STRESSES SIG(XX)	SIG(YY)	SIG(ZZ)	SIG(XY)	SIG(XZ)	SIG(YZ)	PRINCIPAL STRESSES SIG(1)	SIG(2)	SIG(3)
0.0	-0.130+01	0.480+02	0.530+02	-0.180+01	-0.190-02	-0.520-01	-0.400+00	0.530+02	0.480+02	-0.180+01

Comments:

In both cases maximum vertical displacement (elastic) is 1.3 inches. The maximum principle stress SIG(1) is 41 psi at the main axis, and 53 psi at the wheel closest to main axis.

```
CARD INPUT IMAGES FOR PROBLEM 2.

COL. NO.  0    5    10   15   20   25   30   35   40   45   50   55

CARD NO.

     A1.  MESH  EXAMPLE 2.  C130 AIRCRAFT, VISCOELASTIC SOLUTION.
     B1.        66.0    2
     B2.         0.0      -27.0
     B2.        66.0       -3.0
     C1.  VISC     0        12       1.0
     D1.  C130                            0.9
     E1.     1     1
     F1.     1
     A1.  STOP
```

Complete data input for problem 2 is shown above. Superimposed output is shown on the following page at the two points of superposition. Because the solution is viscoelastic displacements and stresses are given as a function of time.

Problem 2: Selected Output

FINAL SUPERIMPOSED DISPLACEMENTS AND STRESSES FOR ALL LOAD CONTRIBUTIONS ARE GIVEN BELOW AT EACH TIME STEP (ONE STEP FOR ELASTIC CASE)

- main axis -

TIME	VERTICAL DISP.	CARTESIAN STRESSES SIG(XX)	SIG(YY)	SIG(ZZ)	SIG(XY)	SIG(XZ)	SIG(YZ)	PRINCIPAL STRESSES SIG(1)	SIG(2)	SIG(3)
0.0	-0.47D+00	0.11D+02	0.15D+02	-0.44D+00	0.14D-16	0.92D-18	-0.27D-03	0.15D+02	0.11D+02	-0.44D+00
1.000	-0.53D+00	0.11D+02	0.15D+02	-0.45D+00	0.14D-16	0.33D-18	-0.11D-02	0.15D+02	0.11D+02	-0.45D+00
2.000	-0.59D+00	0.11D+02	0.15D+02	-0.46D+00	0.14D-16	0.11D-17	-0.19D-02	0.15D+02	0.11D+02	-0.46D+00
3.000	-0.65D+00	0.11D+02	0.15D+02	-0.46D+00	0.14D-16	0.15D-17	-0.26D-02	0.15D+02	0.11D+02	-0.46D+00
4.000	-0.70D+00	0.11D+02	0.14D+02	-0.47D+00	0.14D-16	0.87D-18	-0.33D-02	0.14D+02	0.11D+02	-0.47D+00
5.000	-0.76D+00	0.11D+02	0.14D+02	-0.47D+00	0.14D-16	0.87D-18	-0.39D-02	0.14D+02	0.11D+02	-0.47D+00
6.000	-0.81D+00	0.10D+02	0.14D+02	-0.47D+00	0.14D-16	0.87D-18	-0.45D-02	0.14D+02	0.10D+02	-0.47D+00
7.000	-0.85D+00	0.10D+02	0.14D+02	-0.47D+00	0.14D-16	0.87D-18	-0.50D-02	0.14D+02	0.10D+02	-0.47D+00
8.000	-0.90D+00	0.10D+02	0.14D+02	-0.47D+00	0.14D-16	0.87D-18	-0.55D-02	0.14D+02	0.10D+02	-0.47D+00
9.000	-0.94D+00	0.10D+02	0.14D+02	-0.48D+00	0.14D-16	0.87D-18	-0.59D-02	0.14D+02	0.10D+02	-0.48D+00
10.000	-0.98D+00	0.99D+01	0.14D+02	-0.48D+00	0.14D-16	0.87D-18	-0.63D-02	0.14D+02	0.99D+01	-0.48D+00
11.000	-0.10D+01	0.98D+01	0.13D+02	-0.48D+00	0.14D-16	0.87D-18	-0.67D-02	0.13D+02	0.98D+01	-0.48D+00
12.000	-0.11D+01	0.97D+01	0.13D+02	-0.48D+00	0.14D-16	0.87D-18	-0.70D-02	0.13D+02	0.97D+01	-0.48D+00

FINAL SUPERIMPOSED DISPLACEMENTS AND STRESSES FOR ALL LOAD CONTRIBUTIONS ARE GIVEN BELOW AT EACH TIME STEP (ONE STEP FOR ELASTIC CASE)

- wheel closest to main axis -

TIME	VERTICAL DISP.	CARTESIAN STRESSES SIG(XX)	SIG(YY)	SIG(ZZ)	SIG(XY)	SIG(XZ)	SIG(YZ)	PRINCIPAL STRESSES SIG(1)	SIG(2)	SIG(3)
0.0	-0.46D+00	0.18D+02	0.19D+02	-0.37D+00	-0.36D+00	-0.17D-01	-0.50D-02	0.19D+02	0.18D+02	-0.37D+00
1.000	-0.53D+00	0.18D+02	0.19D+02	-0.37D+00	-0.36D+00	-0.18D-01	-0.61D-02	0.19D+02	0.18D+02	-0.37D+00
2.000	-0.59D+00	0.18D+02	0.19D+02	-0.38D+00	-0.37D+00	-0.18D-01	-0.71D-02	0.19D+02	0.18D+02	-0.38D+00
3.000	-0.65D+00	0.18D+02	0.19D+02	-0.38D+00	-0.37D+00	-0.19D-01	-0.81D-02	0.19D+02	0.18D+02	-0.38D+00
4.000	-0.70D+00	0.18D+02	0.19D+02	-0.38D+00	-0.37D+00	-0.19D-01	-0.89D-02	0.19D+02	0.18D+02	-0.38D+00
5.000	-0.75D+00	0.18D+02	0.19D+02	-0.38D+00	-0.36D+00	-0.20D-01	-0.97D-02	0.19D+02	0.17D+02	-0.38D+00
6.000	-0.80D+00	0.17D+02	0.18D+02	-0.38D+00	-0.36D+00	-0.20D-01	-0.10D-01	0.18D+02	0.17D+02	-0.38D+00
7.000	-0.85D+00	0.17D+02	0.18D+02	-0.38D+00	-0.36D+00	-0.20D-01	-0.11D-01	0.18D+02	0.17D+02	-0.38D+00
8.000	-0.89D+00	0.17D+02	0.18D+02	-0.38D+00	-0.36D+00	-0.21D-01	-0.12D-01	0.18D+02	0.17D+02	-0.38D+00
9.000	-0.93D+00	0.17D+02	0.18D+02	-0.38D+00	-0.36D+00	-0.21D-01	-0.12D-01	0.18D+02	0.17D+02	-0.38D+00
10.000	-0.97D+00	0.17D+02	0.18D+02	-0.38D+00	-0.36D+00	-0.21D-01	-0.13D-01	0.18D+02	0.17D+02	-0.38D+00
11.000	-0.10D+01	0.17D+02	0.18D+02	-0.39D+00	-0.36D+00	-0.22D-01	-0.13D-01	0.18D+02	0.17D+02	-0.39D+00
12.000	-0.11D+01	0.17D+02	0.18D+02	-0.39D+00	-0.35D+00	-0.22D-01	-0.14D-01	0.18D+02	0.16D+02	-0.39D+00

```
CARD INPUT IMAGES FOR PROBLEM 3.

COL. NO.  0    5    10   15   20   25   30   35   40   45   50   55

CARD NO.

        A1.  MESH   EXAMPLE 3. RECTANGULAR LOAD PATTERN, VISCOELASTIC
        B1.           50.0    1
        B2.            0.0      -10.0
        C1.  VISC        0          12         1.0
        D1.  USER          10.0       12.73
        E1.          1    1
        F1.          1
        F2.          5   20000.0      -50.0
        F3.         -40.0        0.0        0.2
        F3.         -20.0        0.0        0.2
        F3.           0.0        0.0        0.2
        F3.          20.0        0.0        0.2
        F3.          40.0        0.0        0.2
        F3.
        A1.  STOP
```

Complete input for problem 3 is shown above. Superimposed output at the center of rectangular load pattern is shown on the following page.

Problem 3: Selected Output

FINAL SUPERIMPOSED DISPLACEMENTS AND STRESSES FOR ALL LOAD CONTRIBUTIONS ARE GIVEN BELOW AT EACH TIME STEP (ONE STEP FOR ELASTIC CASE)

- center of rectangular load pattern -

TIME	VERTICAL DISP.	CARTESIAN STRESSES SIG(XX)	SIG(YY)	SIG(ZZ)	SIG(XY)	SIG(XZ)	SIG(YZ)	PRINCIPAL STRESSES SIG(1)	SIG(2)	SIG(3)
0.0	-0.14D+00	0.14D+02	0.16D+02	0.51D-01	0.0	0.11D-01	0.0	0.16D+02	0.14D+02	0.51D-01
1.000	-0.16D+00	0.14D+02	0.15D+02	0.49D-01	0.0	0.11D-01	0.0	0.15D+02	0.14D+02	0.49D-01
2.000	-0.18D+00	0.13D+02	0.15D+02	0.47D-01	0.0	0.11D-01	0.0	0.15D+02	0.13D+02	0.47D-01
3.000	-0.19D+00	0.13D+02	0.15D+02	0.45D-01	0.0	0.11D-01	0.0	0.15D+02	0.13D+02	0.45D-01
4.000	-0.21D+00	0.13D+02	0.15D+02	0.43D-01	0.0	0.10D-01	0.0	0.15D+02	0.13D+02	0.43D-01
5.000	-0.23D+00	0.13D+02	0.15D+02	0.42D-01	0.0	0.10D-01	0.0	0.15D+02	0.13D+02	0.42D-01
6.000	-0.24D+00	0.13D+02	0.14D+02	0.40D-01	0.0	0.10D-01	0.0	0.14D+02	0.13D+02	0.40D-01
7.000	-0.26D+00	0.13D+02	0.14D+02	0.39D-01	0.0	0.10D-01	0.0	0.14D+02	0.13D+02	0.39D-01
8.000	-0.27D+00	0.12D+02	0.14D+02	0.38D-01	0.0	0.10D-01	0.0	0.14D+02	0.12D+02	0.38D-01
9.000	-0.29D+00	0.12D+02	0.14D+02	0.37D-01	0.0	0.10D-01	0.0	0.14D+02	0.12D+02	0.37D-01
10.000	-0.30D+00	0.12D+02	0.14D+02	0.36D-01	0.0	0.10D-01	0.0	0.14D+02	0.12D+02	0.36D-01
11.000	-0.31D+00	0.12D+02	0.14D+02	0.35D-01	0.0	0.10D-01	0.0	0.14D+02	0.12D+02	0.35D-01
12.000	-0.32D+00	0.12D+02	0.14D+02	0.34D-01	0.0	0.99D-02	0.0	0.14D+02	0.12D+02	0.34D-01

APPENDIX B

PROGRAM VISFIT

Discussed in this appendix is the program VISFIT whose purpose is to compute parameters of viscoelastic functions to conform to experimental creep or relaxation data. The resulting parameters of the viscoelastic function may be used as input for the VISICE program.

Part 1 of this appendix provides a general background discussion on the nature of the problem and Part 2 contains the user instructions and example problems for VISFIT.

Part 1. Nature of the Problem

General: The following background discussion will be focused on general concepts with minimal analytical development (which can be obtained from cited references). Attention is restricted to identification of viscoelastic parameters from one dimensional creep tests, i.e., a constant axial stress (tension or compression) applied to a material specimen resulting in creep strain data as a function of time.

As a side comment, it is noted that the VISFIT program permits identification of either creep data or relaxation data for any simple experimental test with a prescribed stress state or strain state. However, the uniaxial creep test described above is most common and offers a convenient format for discussion. More general applications of VISFIT will be apparent from input instructions.

Objective: The problem statement is; given a set of creep data what are the corresponding relaxation function parameters suitable for input into the VISICE program.

By creep data it is meant a collection of data points (ε_i, t_i) where ε_i is the measured strain at time t_i from a unit of applied constant stress. (If applied stress is not unity the data can be scaled to unity, VISFIT does this automatically).

To meet the above objective, three distinct operations are required:

(1) Curve Fit - Determine a creep function that best represents the creep data.

(2) Invert - Find the inverse of the creep function called the relaxation function. (Young's relaxation function).

(3) Transform - Obtain the bulk and shear relaxation function from Young's modulus relaxation function. (This is a trivial operation if Poisson's ratio is constant).

Each of the above operations are discussed in turn in the context of methods employed in the VISFIT program.

Curve Fitting: The general form of the creep function to represent any set of creep data is:

$$Y(t) = A + Bt + C_1 (1 - e^{-x_1 t}) + C_2 (1 - e^{-x_2 t}) + C_3 (1 - e^{-x_3 t}) \quad \text{(B-1)}$$

where $Y(t)$ = Creep Function.
A = Initial Elastic Response (Constant).
B = Linear Creep Rate at Long Times.
C_i = Coefficient of Exponential Term i.
X_i = Exponent of Exponential Term i (inverse of retardation time).

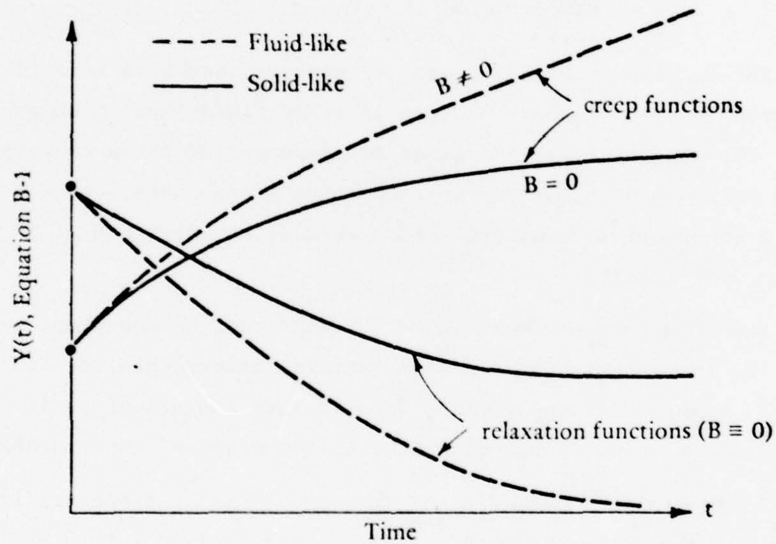
In all cases the parameter A is nonzero, and B is zero if the creep response is solid-like or nonzero if it is fluid-like as shown in Figure B-1. One should strive to use as few exponential terms as possible but still maintain an accurate representation of the data. In practice, more than 3 exponential terms are seldom if ever required and it is most common to use 1 or 2 terms.

Note the form of Equation B-1 is such that the constant parameter A is the initial elastic response (strain) rather than the long-term ($t = \infty$) elastic response. This form is more convenient as it provides the option to specify the parameter A from observed experimental data.

In general the objective of the curve fitting procedure in VISFIT is to determine the parameters A, B, C_i and X_i (i = 1 to 3 maximum) so that the creep function Y(t) "best" matches the experimental data points. This is accomplished by a least-square-error technique developed in detail in Reference (5). Briefly, the technique minimizes net error arising from squared difference of the creep function minus interpolated data function integrated over the entire time domain. As a result, a set of "N" coupled algebraic equations are produced which are solved for the "N" unknown parameters. These equations are nonlinear in X_i and are solved iteratively by a Newton-Raphson procedure (5).

VISFIT permits pre-specifying any subset of the parameters to "known" values so that the "N" unknown values may be less than the actual number

Boundary conditions should not appreciably influence the solution for large plate radius.



B = linear parameter of Equation B-1

Figure B-1. General forms of Equation B-1.

of parameters in the selected creep function. This capability of the program has many useful applications. For example, the initial elastic response A may be predefined. Also, the parameter B may be set to zero if a solid-like response is desired or set to some long term creep rate for fluid-like response.

Perhaps the most useful application of pre-specifying parameters is with regard to the exponents X_i. In this case, the motivation is not because X_i is apriori known from observing experimental data (indeed this is difficult) but rather, the motivation is to merely obtain a converged solution. To appreciate this problem, it must be understood that the N coupled equations are highly nonlinear whenever there are unknown X_i terms. Accordingly, whether or not the Newton-Raphson iteration procedure will converge to a solution is dependent on the initial guesses for each X_i term and the consistency of the creep data input.

In view of the above, the user may have to interact with VISFIT program to obtain a solution. Typically, the first attempt would be to assume each X_i is unknown along with each C_i and perhaps A and B as well. If VISFIT does not indicate a converged solution, estimate some specified value for one of the X_i and try again. If convergence is still not obtained (assuming more than one exponential term) estimate values for all X_i. When this is done the algebraic equations are linear and a converged solution is always obtained. At this point, the user examines the relative error of the creep function representation of the data points (displayed in the VISFIT output). If the fit is not acceptably close, new estimates of the X_i should be assumed and the process repeated. When two or more exponential terms are specified it is good practice to specify $X_2 = 10X_1$, and $X_3 = 100X_1$.

VISFIT output for curve fitting gives the parameters A, B, C_i and X_i for Equation B1. In addition it gives the parameters $\bar{A}$, B, $\bar{C}_i$ and X_i for the same function in a slightly different form:

$$Y(t) = \bar{A} + Bt + \bar{C}_1 e^{-X_1 t} + \bar{C}_2 e^{-X_2 t} + \bar{C}_3 e^{-X_3 t} \quad \text{(B-2)}$$

where: $\bar{A} = A + C_1 + C_2 + C_3$ (B-3)

$$\bar{C}_i = -C_i$$

here the constant term $\bar{A}$ is the long term elastic strain and $\bar{C}_i$ are negative coefficients of exponential terms. Parameters B and X_i are the same values in both forms. The form given by Equation B-2 is more convenient for inversion and corresponds to the desired form of the relaxation function for VISICE.

Inversion: To obtain the relaxation function corresponding to the creep function, Equation B-2, the following convolution identity must be satisfied:

$$E(t)\, Y(0) + \int_o^t E(t - \tau) \frac{\partial Y}{\partial \tau} d\tau = 1 \quad \text{(B-4)}$$

with: $E(t) = A^* + C_1^* e^{-X_1^*, t} + C_2^* e^{-X_2^*, t} + C_3^* e^{-X_3^*, t}$ (B-5)

where: $E(t)$ = Relaxation function (Young's modulus)

A^* = Modulus at infinite time

C_1^* = Modulus coefficients of exponential term i

X_i^* = Exponent of exponential term i (inverse of relaxation time)

Satisfaction of Equation B-4 provides a set of nonlinear algebraic equations that relate relaxation parameters A^*, C_i^*, and X_i^* to the creep parameters $\bar{A}$, B, $\bar{C}_i$, and X_i. These equations have been solved exactly using the procedure in Reference (5) and is coded in the VISFIT program. Because of the exact solution procedure no convergence problems are encountered.

If the creep function is fluid-like ($B \neq 0$), the corresponding solid-like relaxation function is of the form, $A^* = 0$ and an additional exponential term is applied. On the other hand, if the creep function is solid-like ($B = 0$), the corresponding solid-like relaxation is of the form, $A^* \neq 0$

and both functions have the same number of exponential terms. VISFIT automatically outputs the proper parameters for the inverse function.

Transforming: The relaxation function given by Equation B-5 represents a Young's modulus function which must be converted to bulk and shear relaxation functions denoted by:

$$K(t) = K_o + K_1 e^{-t/\alpha_1} + K_2\, e^{-t/\alpha_2} + K_3\, e^{-t/\alpha_3} \qquad \text{(B-6)}$$

$$G(t) = G_o + G_1\, e^{-t/\beta_1} + G_2\, e^{-t/\beta_2} + G_3\, e^{-t/\beta_3} \qquad \text{(B-7)}$$

where K(t) and G(t) are the bulk and shear relaxation functions used as input to the VISICE program as previously described by Equations 3 and 4 in the main report.

If Poisson's ratio, v, is observed (or assumed) constant (i.e., axial and lateral strains creep in proportion), the bulk and shear conversion is exactly like an elastic solid. For bulk we have:

$$K(t) = \phi(v)\; E(t)$$

where: $\phi(v) = 1/3\;(1 - 2v)$

Thus; $K_o = \phi(v)\; A^*$

$$K_i = \phi(v)\; C_i^*$$

$$\alpha_i = \frac{1}{X_i}$$

And for shear we have:

$$G(t) = \theta(v)\; E(t)$$

where: $\theta(v) = 1/2\;(1 + v)$

Thus; $G_o = \theta(v)\; A^*$

$$G_i = \theta(v)\; C_i^*$$

$$\beta_i = \frac{1}{X_i}$$

Note the relaxation times for bulk and shear (α_i and β_i) are identical and given by the inverse X_i. Both X_i and its inverse are printed out by VISFIT.

If Poisson's ratio is not constant, dual curve fitting procedures must be used as described in Reference (5). However, for many materials including sea ice, a constant Poisson's ratio is adequate.

PART II. INPUT INSTRUCTIONS FOR VISFIT

VISFIT provides the capability of curve fitting creep data (or relaxation data) to a creep function (or relaxation function) and inverting the creep function to a relaxation function (or visa-versa). A maximum of three exponential terms may be used to represent the creep/relaxation function and each parameter of the function may be specified (fixed) or unknown.

Input data cards are grouped as follows:

A. Executive control card.

B. Identification of viscoelastic parameters for curve fit.

C. Input data from experimental creep or relaxation test.

D. Parameter input for inversion only.

Card group D is only required when inverting some known function that was not produced by the curve fitting procedure (Cards A, B, and C).

INPUT INSTRUCTIONS

A. Executive Control Card and Heading

Card A1 (I4, 17A4)

Columns	Variable	Entry Description	Note
01-04	MODE	Mode of program operation: = 0, Curve fit and invert = 1, Curve fit only = 2, Invert only = -1, Program stop	(1)
05-72	HED(17)	Descriptive title of problem	(2)

- Go to Card B1 -

(Unless MODE = 2 then go to Card D1)

Notes:

(1) Mode of operation specified whether curve fitting, or inverting, or both are to be considered by the program. For MODE = 0 or 1 only card sets A, B, C need be supplied. For MODE = 2 only card set D is supplied. Problems may be run back-to-back, to terminate set MODE = -1.

(2) User defined heading to printed with results.

B. Identification of Viscoelastic Function Parameters

Card B1 (2I5, F10.0) Master control for curve fitting.

Columns	Variable	Entry Description	Notes
01-05	NUMEXP	Number of exponential terms desired, Range: 0 to 3.	(1)
06-10	LIMIT	Number of Newton-Raphson iterations, Default = 20.	(2)
11-20	TOLER	Tolerance of relative convergence, Default = 0.01 (1% error).	(3)

Notes:

(1) In general one should limit the number of exponential terms to few as possible while still achieving a proper representation of the data (trial and error). Usually 1 or 2 terms are sufficient.

(2) Experience has shown that if convergence doesn't occur in 20 iterations, convergence will probably not occur at all.

(3) Tolerance of convergence is a measure of the absolute change in the viscoelastic parameter values in two successive iterations. It is not a measure of how close the viscoelastic function matches the input data.

Card B2 (2(I5, F10.0)) Constant and Linear Viscoelastic Parameters

Columns	Variable	Entry Description	Notes
01-05	NA	Control for constant parameter A; = 0, implies A is specified = 1, implies A is unknown	(1)
06-15	A	Value of constant parameter A; = specified value if NA = 0 = initial guess if NA = 1	(2)
16-20	NB	Control for linear parameter B; = 0, implies B is specified = 1, implies B is unknown	(3)
21-30	B	Value of linear parameter B; = specified value if NB = 0 = initial guess if NB = 1	(4)

Notes:

(1,2) The parameter A is the initial elastic response. Usually it is desirable to specify this as a fixed value (NA = 0) based on the observed initial elastic response.

(3,4) The parameter B is the steady creep rate at long times. As an initial guess, the slope of creep curve at long times may be used. If the model is to be solid-like B must be specified zero. (For relaxation functions set B = 0.0).

Card B3 (2(I5, F10.0)) Exponential Viscoelastic Parameters, Repeat this card NUMEXP times

Columns	Variable	Entry Description	Notes
01-05	NC(I)	Control for exponential coefficient, C(I); = 0, implies C(I) is specified = 1, implies C(I) is unknown	(1)
06-15	C(I)	Value of exponential coefficient, C(I); = specified value if NC(I) = 0 = initial guess if NC(I) = 1	(2)
16-20	NX(I)	Control for exponent, X(I); = 0, implies X(I) is specified = 1, implies X(I) is unknown	(3)
21-30	X(I)	Value of exponent X(I); = specified value if NX(I) = 0 = initial guess if NX(I) = 1	(4)

Notes:

(1,2) Usually, the coefficients C(I) are unknown and are determined by program (NC(I) = 1). Initial guesses for C(I) are automatically provided by the program if no initial guess is input by the user.

(3,4) Unknown exponents X(I) are the source of all convergence problems. If a converged solution is not obtained with NX(I) = 1, try specifying values of X(I) in a trial and error process. For more than one exponential term, it is good practice to specify each X(I) an order of magnitude greater than the previous.

C. Input Data from Experimental Creep/Relaxation Tests

Card C1 (I5, F10.0) Master Control of Input Data

Columns	Variable	Entry Description	Notes
01-05	NDATA	Number of data points input (25 maximum)	
06-15	YSCALE	Scale factor to be multiplied by creep/relaxation data	(1)

Card C2 (2F10.0) Time and Creep/Relaxation Data
Repeat this card NDATA times

Columns	Variable	Entry Description	Notes
01-10	T(I)	Time of data point I	(2)
11-20	Y(I)	Data value of creep/relaxation test at time T(I)	(3)

- Data Input is Complete for MODE = 0 or 1 -

Notes:

(1) YSCALE allows normalizing the input data Y(I) for unit loading, e.g., if the creep data represents strains for 10 psi loading pressure, set YSCALE = 0.1.

(2,3) The first data point (T(1), Y(1)) is assumed to be T(1) = 0.0 and Y(1) = initial elastic response from step loading. If T(1) $\neq$ 0.0 all time data is shifted so that T(1) = 0.0. Time data should be ascending order, but there is no restriction for equal time increments.

D. Input Data for Inversion of a Viscoelastic Function (MODE = 2 only)

Card D1 (2F10.0) Constant and Linear Viscoelastic Parameters

Columns	Variable	Entry Description	Notes
01-10	$\bar{A}$	Value of constant parameter representing viscoelastic function value at time = ∞	(1)
11-20	B	Value of linear parameter for fluid-like creep function	(2)

Card D2 (6F10.0) Exponential Viscoelastic Parameters (3-term maximum)

Columns	Variable	Entry Description	Notes
01-10	$\bar{C}(1)$	Coefficient of first exponential term	(3)
11-20	X(1)	Exponent of first exponential term	(4)
21-30	$\bar{C}(2)$	Coefficient of second exponential term	
31-40	X(2)	Exponent of second exponential term	
41-50	$\bar{C}(3)$	Coefficient of third exponential term	
51-60	X(3)	Exponent of third exponential term	

- Data Input is Complete for Inversion, MODE = 2 -

See Notes on Next Page

Notes for Card Set D:

General: Inversion is performed from a specified creep function to a relaxation function, or a specified relaxation function to a creep function. The function to be inverted is of the form:

$$Y(t) = \bar{A} + Bt + \bar{C}_1 e^{-x_1 t} + \bar{C}_2 e^{-x_2 t} + \bar{C}_3 e^{-x_3 t}$$

The resulting inverted function is of the same general form, but some parameters may be zero or added (see below).

(1,2) Specification of input parameters $\bar{A}$ and B (i.e., zero or nonzero) dictate the nature of the input function and the form of the resulting inverse function as shown in Table A1.

(3,4) Each nonzero entry of the coefficients $\bar{C}$(I) and exponent X(I) identifies how many exponential terms are being considered in the input function, 3 maximum. For example, two exponential terms would require input values for C(1), X(1), C(2) and X(2) (C(3) = X(3) = 0.0).

TABLE A1. Function Forms

Input Parameters		Input Function	Output Function	Output Parameters		Output: No. of exponential
$\bar{A}$	B	Type	Type	A*	B	Terms
0	0	fluid-like relaxation	fluid-like creep	Ø	Ø	less one term
Ø	Ø	Fluid-like creep	fluid-like relaxation	0	0	plus one term
Ø	0	solid-like either relaxation or creep.	solid-like either creep or relaxation.	Ø	0	no change
0	Ø	*** Not Valid Form ***				

Example Problem for VISFIT

Problem Description: The set of creep data shown in Table B2 (25 points) is to be represented by a creep function of the form:

$$Y(t) = A + C_1(1-e^{-X_1 t})$$

Since this form is solid-like, the linear parameter B is specified as zero in the VISIFT input (NB = 0). Parameters A, C_1, and X_1 are taken as unknowns, and all the default options of VISFIT are employed.

TABLE B2. Creep Data (Hypothetical)

Time	Creep Strain	Time	Creep Strain
0.00	0.500	0.13	0.553
0.01	0.504	0.14	0.557
0.02	0.509	0.15	0.560
0.03	0.514	0.16	0.563
0.04	0.518	0.17	0.566
0.05	0.523	0.18	0.569
0.06	0.527	0.19	0.572
0.07	0.531	0.20	0.575
0.08	0.535	0.30	0.598
0.09	0.539	0.40	0.616
0.10	0.543	0.50	0.629
0.11	0.546	0.70	0.646

Problem Solution: Using MODE = 0, VISFIT gives the creep the function as:

$$Y(t) = 0.4996 + 0.1658\,(1.0 - e^{-2.996t})$$

or in the alternative form:

$$Y(t) = 0.6654 - 0.1658e^{-2.996t}$$

The corresponding relaxation function (i.e., inverse function) is:

$$Y^*(t) = 1.5028 + 0.4986e^{-3.990t}$$

For input into the VISICE program, the relaxation function is converted into bulk and shear relaxation functions as described in previous sections.